Mathematics

(Chapter – 2) (Relations and Functions)

(Class – XI)

Exercise 2.1

Question 1:

If

$\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$, find the values of *x* and *y*.

Answer 1:

It is given that

$$\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right).$$

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$ $\quad y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3}$ $\quad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow x=2$ $\quad \Rightarrow y=1$

∴ $x = 2$ and $y = 1$

Question 2:

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Answer 2:

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

$\Rightarrow$ Number of elements in set B = 3

Number of elements in (A × B)
= (Number of elements in A) × (Number of elements in B)
= 3 × 3 = 9

Thus, the number of elements in (A × B) is 9.

Question 3:
If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Answer 3:
G = {7, 8} and H = {5, 4, 2}
We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as P × Q = {(p, q): $p \in$ P, $q \in$ Q}
∴ G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}
H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

Question 4:
State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}.

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \in$ A and $y \in$ B.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.

Answer 4:
(i) False
If P = {m, n} and Q = {n, m}, then
P × Q = {(m, m), (m, n), (n, m), (n, n)}
(ii) True
(iii) True

Question 5:
If A = {−1, 1}, find A × A × A.

Answer 5:
It is known that for any non-empty set A, A × A × A is defined as
A × A × A = {(*a*, *b*, *c*): *a*, *b*, *c* ∈ A}

It is given that A = {−1, 1}

∴ A × A × A = {(−1, −1, −1), (−1, −1, 1), (−1, 1, −1), (−1, 1, 1), (1, −1, −1), (1, −1, 1), (1, 1, −1), (1, 1, 1)}

Question 6:
If A × B = {(*a*, *x*), (*a*, *y*), (*b*, *x*), (*b*, *y*)}. Find A and B.

Answer 6:
It is given that A × B = {(*a*, *x*), (*a*, *y*), (*b*, *x*), (*b*, *y*)}
We know that the Cartesian product of two non-empty sets P and Q is defined as P × Q = {(*p*, *q*): *p* ∈ P, *q* ∈ Q}

∴ A is the set of all first elements and B is the set of all second elements.

Thus, A = {*a*, *b*} and B = {*x*, *y*}

Question 7:
Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that
(i) A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) A × C is a subset of B × D

Answer 7:
(i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)

We have B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ

∴ L.H.S. = A × (B ∩ C) = A × Φ = Φ
A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

∴ R.H.S. = (A × B) ∩ (A × C) = Φ

∴ L.H.S. = R.H.S

Hence, A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) To verify: A × C is a subset of B × D

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
A × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

We can observe that all the elements of set A × C are the elements of set B × D. Therefore, A × C is a subset of B × D.

Question 8:
Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Answer 8:
A = {1, 2} and B = {3, 4}

∴ A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}
⇒ n(A × B) = 4

We know that if C is a set with n(C) = m, then n[P(C)] = 2^m.

Therefore, the set A × B has 2^4 = 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

Question 9:

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in A × B, find A and B, where x, y and z are distinct elements.

Answer 9:

It is given that $n(A) = 3$ and $n(B) = 2$; and $(x, 1), (y, 2), (z, 1)$ are in A×B.

We know that
A = Set of first elements of the ordered pair elements of A × B
B = Set of second elements of the ordered pair elements of A × B.

∴ x, y, and z are the elements of A; and 1 and 2 are the elements of B.

Since $n(A) = 3$ and $n(B) = 2$,

it is clear that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

Question 10:

The Cartesian product A × A has 9 elements among which are found (−1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

Answer 10:

We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

$\therefore n(A \times A) = n(A) \times n(A)$

It is given that $n(A \times A) = 9$

$\therefore n(A) \times n(A) = 9$
$\Rightarrow n(A) = 3$

The ordered pairs (−1, 0) and (0, 1) are two of the nine elements of A×A.

We know that $A \times A = \{(a, a): a \in A\}$. Therefore, −1, 0, and 1 are elements of A.
Since $n(A) = 3$, it is clear that A = {−1, 0, 1}.
The remaining elements of set A × A are (−1, −1), (−1, 1), (0, −1), (0, 0), (1, −1), (1, 0), and (1, 1).

Mathematics

(Chapter – 2) (Relations and Functions)

(Class – XI)

Exercise 2.2

Question 1:
Let A = {1, 2, 3… 14}. Define a relation R from A to A by R = {(x, y): $3x - y = 0$, where $x, y \in$ A}. Write down its domain, codomain and range.

Answer 1:
The relation R from A to A is given as R = {(x, y): $3x - y = 0$, where $x, y \in$ A}

i.e., R = {(x, y): $3x = y$, where $x, y \in$ A}
∴ R = {(1, 3), (2, 6), (3, 9), (4, 12)}

The domain of R is the set of all first elements of the ordered pairs in the relation.
∴ Domain of R = {1, 2, 3, 4}
The whole set A is the codomain of the relation R.
∴ Codomain of R = A = {1, 2, 3… 14}
The range of R is the set of all second elements of the ordered pairs in the relation.
∴Range of R = {3, 6, 9, 12}

Question 2:
Define a relation R on the set **N** of natural numbers by R = {(x, y): $y = x + 5$, x is a natural number less than 4; $x, y \in$ **N**}. Depict this relationship using roster form. Write down the domain and the range.

Answer 2:
R = {(x, y): $y = x + 5$, x is a natural number less than 4, $x, y \in$ **N**}
The natural numbers less than 4 are 1, 2, and 3.
∴ R = {(1, 6), (2, 7), (3, 8)}
The domain of R is the set of all first elements of the ordered pairs in the relation.
∴ Domain of R = {1, 2, 3}
The range of R is the set of all second elements of the ordered pairs in the relation. ∴ Range of R = {6, 7, 8}

Question 3:

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(*x*, *y*): the difference between *x* and *y* is odd; *x* ∈ A, *y* ∈ B}. Write R in roster form.

Answer 3:

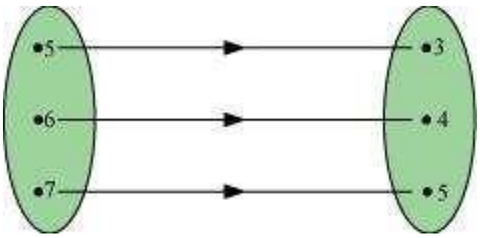
A = {1, 2, 3, 5} and B = {4, 6, 9}

R = {(*x*, *y*): the difference between *x* and *y* is odd; *x* ∈ A, *y* ∈ B}

∴ R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Question 4:

The given figure shows a relationship between the sets P and Q. write this relation **(i)** in set-builder form **(ii)** in roster form. What is its domain and range?



Answer 4:

According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) R = {(*x*, *y*): *y* = *x* − 2; *x* ∈ P} or R = {(*x*, *y*): *y* = *x* − 2 for *x* = 5, 6, 7}

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

Question 5:

Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by {(*a*, *b*): *a*, *b* ∈ A, *b* is exactly divisible by *a*}.

(i) Write R in roster form
(ii) Find the domain of R
(iii) Find the range of R.

Answer 5:

A = {1, 2, 3, 4, 6}, R = {(*a*, *b*): *a*, *b* ∈ A, *b* is exactly divisible by *a*}
(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}
(ii) Domain of R = {1, 2, 3, 4, 6}
(iii) Range of R = {1, 2, 3, 4, 6}

Question 6:

Determine the domain and range of the relation R defined by
R = {(x, $x + 5$): x ∈ {0, 1, 2, 3, 4, 5}}.

Answer 6:

R = {(x, $x + 5$): x ∈ {0, 1, 2, 3, 4, 5}}
∴ R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
∴ Domain of R = {0, 1, 2, 3, 4, 5}
Range of R = {5, 6, 7, 8, 9, 10}

Question 7:

Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.

Answer 7:

R = {(x, x^3): x is a prime number less than 10} The prime numbers less than 10 are 2, 3, 5, and 7.

∴ R = {(2, 8), (3, 27), (5, 125), (7, 343)}

Question 8:

Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Answer 8:

It is given that A = {x, y, z} and B = {1, 2}.

∴ A × B = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}

Since n(A × B) = 6, the number of subsets of A × B is 2^6.

Therefore, the number of relations from A to B is 2^6.

Question 9:

Let R be the relation on **Z** defined by R = {(a, b): a, b ∈ **Z**, $a - b$ is an integer}. Find the domain and range of R.

Answer 9:

R = {(a, b): a, b ∈ **Z**, $a - b$ is an integer}

It is known that the difference between any two integers is always an integer.

∴ Domain of R = **Z**

Range of R = **Z**

Mathematics

(Chapter – 2) (Relations and Functions)

(Class – XI)

Exercise 2.3

Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
(iii) {(1, 3), (1, 5), (2, 5)}

Answer 1:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function. Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.
Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}
(iii) {(1, 3), (1, 5), (2, 5)}
Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:

Find the domain and range of the following real function:
(i) $f(x) = -|x|$
(ii) $f(x) = \sqrt{9 - x^2}$

Answer 2:

(i) $f(x) = -|x|$, $x \in$ R

We know that $|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$

$$\therefore\ f(x) = -|x| = \begin{cases} -x, & if\ x \geq 0 \\ x, & if\ x < 0 \end{cases}$$

Since $f(x)$ is defined for $x \in$ **R**, the domain of f is **R.**

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.
∴ The range of f is $(-\infty, 0]$.

(ii) $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to −3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \le x \le 3\}$ or [−3, 3].
For any value of x such that $-3 \le x \le 3$, the value of $f(x)$ will lie between 0 and 3. ∴The range of $f(x)$ is $\{x: 0 \le x \le 3\}$ or [0, 3].

Question 3:
A function f is defined by $f(x) = 2x - 5$. Write down the values of
(i) $f(0)$,
(ii) $f(7)$,
(iii) $f(-3)$

Answer 3:
The given function is $f(x) = 2x - 5$.
Therefore,
(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$
(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$
(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question 4:
The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $f(C) = \frac{9C}{5} + 32$. Find

(i) t (0) **(ii)** t (28) **(iii)** t (−10)
(iv) The value of C, when $t(C) = 212$

Answer 4:
The given function is $f(C) = \frac{9C}{5} + 32$.
Therefore,

(i) $t(0)=\frac{9\times 0}{5}+32=0+32=32$

(ii) $t(28)=\frac{9\times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}$

(iii) $t(-10)=\frac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$

(iv) It is given that $t(C) = 212$

$$\therefore 212=\frac{9C}{5}+32$$
$$\Rightarrow \frac{9C}{5}=212-32$$
$$\Rightarrow \frac{9C}{5}=180$$
$$\Rightarrow 9C=180\times 5$$
$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Thus, the value of t, when $t(C) = 212$, is 100.

Question 5:

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$.

(ii) $f(x) = x^2 + 2$, x, is a real number.

(iii) $f(x) = x$, x is a real number

Answer 5:

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	**0.01**	**0.1**	**0.9**	**1**	**2**	**2.5**	**4**	**5**	**…**
***f*(*x*)**	1.97	1.7	–0.7	–1	–4	–5.5	–10	–13	…

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.
i.e., range of $f = (-\infty, 2)$

Alter:
Let $x > 0$
$\Rightarrow 3x > 0$
$\Rightarrow 2 - 3x < 2$
$\Rightarrow f(x) < 2$
$\therefore$Range of $f = (-\infty, 2)$
(ii) $f(x) = x^2 + 2$, x, is a real number
The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		…
$f(x)$	2	2.09	2.64	3	6	11		…..

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.
i.e., range of $f = [2, \infty)$

Alter:
Let x be any real number. Accordingly,
$x^2 \geq 0$
$\Rightarrow x^2 + 2 \geq 0 + 2$
$\Rightarrow x^2 + 2 \geq 2$
$\Rightarrow f(x) \geq 2$
$\therefore$ Range of $f = [2, \infty)$
(iii) $f(x) = x$, x is a real number
It is clear that the range of f is the set of all real numbers. $\therefore$ Range of f = **R**

Mathematics

(Chapter – 2) (Relations and Functions)

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Miscellaneous Exercise on Chapter 2

Question 1:

The relation *f* is defined by $f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$

The relation *g* is defined by $g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$

Show that *f* is a function and *g* is not a function.

Answer 1:

The relation *f* is defined as

$$f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$$

It is observed that for

$0 \le x < 3$, $f(x) = x^2$

$3 < x \le 10$, $f(x) = 3x$

Also, at $x = 3$, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$ i.e., at $x = 3$, $f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique. Thus, the given relation is a function.

The relation *g* is defined as

$$g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$$

It can be observed that for $x = 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

Question 2:

If $f(x) = x^2$, find. $\frac{f(1.1)-f(1)}{(1.1-1)}$

Answer 2:

$$f(x)=x^2$$

$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-(1)^2}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$$

Question 3:

Find the domain of the function $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

Answer 3:

The given function is $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function f is defined for all real numbers except at x = 6 and x = 2. Hence, the domain of f is **R** – {2, 6}.

Question 4:

Find the domain and the range of the real function f defined by $f(x)=\sqrt{(x-1)}$

Answer 4:

The given real function is $f(x)=\sqrt{(x-1)}$

It can be seen that $\sqrt{(x-1)}$ is defined for $x \geq 1$.

Therefore, the domain of *f* is the set of all real numbers greater than or equal to 1 i.e., the domain of *f* = $[1, \infty)$.

As $x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow \sqrt{(x-1)} \geq 0$
Therefore, the range of *f* is the set of all real numbers greater than or equal to 0 i.e., the range of *f* = $[0, \infty)$.

Question 5:
Find the domain and the range of the real function *f* defined by
$f(x) = |x - 1|$.

Answer 5:
The given real function is $f(x) = |x - 1|$.
It is clear that $|x - 1|$ is defined for all real numbers.
∴ Domain of *f* = **R**
Also, for $x \in \mathbf{R}$, $|x - 1|$ assumes all real numbers.
Hence, the range of *f* is the set of all non-negative real numbers.

Question 6:
Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$

be a function from **R** into **R**. Determine the range of *f*.

Answer 6:

$$f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$$
$$= \left\{(0, 0), \left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{17}\right), \ldots\right\}$$

The range of *f* is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.
[Denominator is greater numerator] Thus, range of *f* = [0, 1)

Question 7:

Let f, g: **R** → **R** be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$.

Answer 7:

f, g: **R** → **R** is defined as $f(x) = x + 1$, $g(x) = 2x - 3$

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$\therefore (f + g)(x) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$

$\therefore (f - g)(x) = -x + 4$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0,\ x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3},\ 2x - 3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3},\ x \neq \frac{3}{2}$$

Question 8:

Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from **Z** to **Z** defined by $f(x) = ax + b$, for some integers a, b. Determine a, b.

Answer 8:

$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ and $f(x) = ax + b$

$(1, 1) \in f \quad \Rightarrow f(1) = 1 \quad \Rightarrow a \times 1 + b = 1$

$\Rightarrow a + b = 1$

$(0, -1) \in f \quad \Rightarrow f(0) = -1 \quad \Rightarrow a \times 0 + b = -1$

$\Rightarrow b = -1$

On substituting $b = -1$ in $a + b = 1$,

We obtain $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$. Thus, the respective values of a and b are 2 and -1.

Question 9:

Let R be a relation from **N** to **N** defined by R = $\{(a, b): a, b \in \mathbf{N}$ and $a = b^2\}$. Are the following true?

(i) $(a, a) \in$ R, for all $a \in \mathbf{N}$

(ii) $(a, b) \in$ R, implies $(b, a) \in$ R

(iii) $(a, b) \in$ R, $(b, c) \in$ R implies $(a, c) \in$ R.

Justify your answer in each case.

Answer 9:

R = $\{(a, b): a, b \in \mathbf{N}$ and $a = b^2\}$

(i) It can be seen that $2 \in \mathbf{N}$; however, $2 \neq 2^2 = 4$.

Therefore, the statement "$(a, a) \in$ R, for all $a \in \mathbf{N}$" is not true.

(ii) It can be seen that $(9, 3) \in \mathbf{N}$ because $9, 3 \in \mathbf{N}$ and $9 = 3^2$. Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin \mathbf{N}$

Therefore, the statement "$(a, b) \in$ R, implies $(b, a) \in$ R" is not true.

(iii) It can be seen that $(9, 3) \in$ R, $(16, 4) \in$ R because $9, 3, 16, 4 \in \mathbf{N}$ and $9 = 3^2$ and $16 = 4^2$.

Now, $9 \neq 4^2 = 16$; therefore, $(9, 4) \notin \mathbf{N}$

Therefore, the statement "$(a, b) \in$ R, $(b, c) \in$ R implies $(a, c) \in$ R" is not true.

Question 10:

Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and *f* = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?

(i) *f* is a relation from A to B

(ii) *f* is a function from A to B.

Justify your answer in each case.

Answer 10:

A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}

∴ A × B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11),
(2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4,
11), (4, 15), (4, 16)}

It is given that *f* = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B.

It is observed that *f* is a subset of A × B.

Thus, *f* is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation *f* is not a function.

Question 11:

Let *f* be the subset of **Z** × **Z** defined by *f* = {(*ab*, *a* + *b*): *a*, *b* ∈ **Z**}. Is *f* a function from **Z** to **Z**: justify your answer.

Answer 11:

The relation *f* is defined as *f* = {(*ab*, *a* + *b*): *a*, *b* ∈ **Z**}

We know that a relation *f* from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since 2, 6, −2, −6 ∈ **Z**, (2 × 6, 2 + 6), (−2 × −6, −2 +
(−6)) ∈ *f* i.e., (12, 8), (12, −8) ∈ *f*

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and −8. Thus, relation *f* is not a function.

Question 12:

Let A = {9, 10, 11, 12, 13} and let f: A → **N** be defined by $f(n)$ = the highest prime factor of n. Find the range of f.

Answer 12:

A = {9, 10, 11, 12, 13} f: A → **N** is defined as $f(n)$ = The highest prime factor of n

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

∴ $f(9)$ = The highest prime factor of 9 = 3

$f(10)$ = The highest prime factor of 10 = 5

$f(11)$ = The highest prime factor of 11 = 11

$f(12)$ = The highest prime factor of 12 = 3

$f(13)$ = The highest prime factor of 13 = 13

The range of f is the set of all $f(n)$, where $n \in$ A.

∴ Range of f = {3, 5, 11, 13}